

Curriculum Focal Points and Connections for Grade 6

The set of three curriculum focal points and related connections for mathematics in grade 6 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 6 Curriculum Focal Points	Connections to the Focal Points
<p>Number and Operations: Developing an understanding of and fluency with multiplication and division of fractions and decimals</p> <p>Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement.</p>	<p>Number and Operations: Students' work in dividing fractions shows them that they can express the result of dividing two whole numbers as a fraction (viewed as parts of a whole). Students then extend their work in grade 5 with division of whole numbers to give mixed number and decimal solutions to division problems with whole numbers. They recognize that ratio tables not only derive from rows in the multiplication table but also connect with equivalent fractions. Students distinguish multiplicative comparisons from additive comparisons.</p>
<p>Number and Operations: Connecting ratio and rate to multiplication and division</p> <p>Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., "If 5 items cost \$3.75 and all items are the same price, then I can find the cost of 12 items by first dividing \$3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12"). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates.</p>	<p>Algebra: Students use the commutative, associative, and distributive properties to show that two expressions are equivalent. They also illustrate properties of operations by showing that two expressions are equivalent in a given context (e.g., determining the area in two different ways for a rectangle whose dimensions are $x + 3$ by 5). Sequences, including those that arise in the context of finding possible rules for patterns of figures or stacks of objects, provide opportunities for students to develop formulas.</p>
<p>Algebra: Writing, interpreting, and using mathematical expressions and equations</p> <p>Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that make the equation true. They solve simple one-step equations by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation. They construct and analyze tables (e.g., to show quantities that are in equivalent ratios), and they use equations to describe simple relationships (such as $3x = y$) shown in a table.</p>	<p>Measurement and Geometry: Problems that involve areas and volumes, calling on students to find areas or volumes from lengths or to find lengths from volumes or areas and lengths, are especially appropriate. These problems extend the students' work in grade 5 on area and volume and provide a context for applying new work with equations.</p>

Curriculum Focal Points and Connections for Grade 7

The set of three curriculum focal points and related connections for mathematics in grade 7 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 7 Curriculum Focal Points	Connections to the Focal Points
<p>Number and Operations and Algebra and Geometry: Developing an understanding of and applying proportionality, including similarity</p> <p>Students extend their work with ratios to develop an understanding of proportionality that they apply to solve single and multistep problems in numerous contexts. They use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease. They also solve problems about similar objects (including figures) by using scale factors that relate corresponding lengths of the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and identify the unit rate as the slope of the related line. They distinguish proportional relationships ($y/x = k$, or $y = kx$) from other relationships, including inverse proportionality ($xy = k$, or $y = k/x$).</p>	<p>Measurement and Geometry: Students connect their work on proportionality with their work on area and volume by investigating similar objects. They understand that if a scale factor describes how corresponding lengths in two similar objects are related, then the square of the scale factor describes how corresponding areas are related, and the cube of the scale factor describes how corresponding volumes are related. Students apply their work on proportionality to measurement in different contexts, including converting among different units of measurement to solve problems involving rates such as motion at a constant speed. They also apply proportionality when they work with the circumference, radius, and diameter of a circle; when they find the area of a sector of a circle; and when they make scale drawings.</p>
<p>Measurement and Geometry and Algebra: Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes</p> <p>By decomposing two- and three-dimensional shapes into smaller, component shapes, students find surface areas and develop and justify formulas for the surface areas and volumes of prisms and cylinders. As students decompose prisms and cylinders by slicing them, they develop and understand formulas for their volumes ($Volume = Area\ of\ base \times Height$). They apply these formulas in problem solving to determine volumes of prisms and cylinders. Students see that the formula for the area of a circle is plausible by decomposing a circle into a number of wedges and rearranging them into a shape that approximates a parallelogram. They select appropriate two- and three-dimensional shapes to model real-world situations and solve a variety of problems (including multistep problems) involving surface areas, areas and circumferences of circles, and volumes of prisms and cylinders.</p>	<p>Number and Operations: In grade 4, students used equivalent fractions to determine the decimal representations of fractions that they could represent with terminating decimals. Students now use division to express any fraction as a decimal, including fractions that they must represent with infinite decimals. They find this method useful when working with proportions, especially those involving percents. Students connect their work with dividing fractions to solving equations of the form $ax = b$, where a and b are fractions. Students continue to develop their understanding of multiplication and division and the structure of numbers by determining if a counting number greater than 1 is a prime, and if it is not, by factoring it into a product of primes.</p>
<p>Number and Operations and Algebra: Developing an understanding of operations on all rational numbers and solving linear equations</p> <p>Students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative integers. By applying properties of arithmetic and considering negative numbers in everyday contexts (e.g., situations of owing money or measuring elevations above and below sea level), students explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense. They use the arithmetic of rational numbers as they formulate and solve linear equations in one variable and use these equations to solve problems. Students make strategic choices of procedures to solve linear equations in one variable and implement them efficiently, understanding that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.</p>	<p>Data Analysis: Students use proportions to make estimates relating to a population on the basis of a sample. They apply percentages to make and interpret histograms and circle graphs.</p> <p>Probability: Students understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Students use theoretical probability and proportions to make approximate predictions.</p>

Curriculum Focal Points and Connections for Grade 8

The set of three curriculum focal points and related connections for mathematics in grade 8 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 8 Curriculum Focal Points	Connections to the Focal Points
<p>Algebra: Analyzing and representing linear functions and solving linear equations and systems of linear equations</p> <p>Students use linear functions, linear equations, and systems of linear equations to represent, analyze, and solve a variety of problems. They recognize a proportion ($y/x = k$, or $y = kx$) as a special case of a linear equation of the form $y = mx + b$, understanding that the constant of proportionality (k) is the slope and the resulting graph is a line through the origin. Students understand that the slope (m) of a line is a constant rate of change, so if the input, or x-coordinate, changes by a specific amount, a, the output, or y-coordinate, changes by the amount ma. Students translate among verbal, tabular, graphical, and algebraic representations of functions (recognizing that tabular and graphical representations are usually only partial representations), and they describe how such aspects of a function as slope and y-intercept appear in different representations. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines that intersect, are parallel, or are the same line, in the plane. Students use linear equations, systems of linear equations, linear functions, and their understanding of the slope of a line to analyze situations and solve problems.</p>	<p>Algebra: Students encounter some nonlinear functions (such as the inverse proportions that they studied in grade 7 as well as basic quadratic and exponential functions) whose rates of change contrast with the constant rate of change of linear functions. They view arithmetic sequences, including those arising from patterns or problems, as linear functions whose inputs are counting numbers. They apply ideas about linear functions to solve problems involving rates such as motion at a constant speed.</p> <p>Geometry: Given a line in a coordinate plane, students understand that all “slope triangles”—triangles created by a vertical “rise” line segment (showing the change in y), a horizontal “run” line segment (showing the change in x), and a segment of the line itself—are similar. They also understand the relationship of these similar triangles to the constant slope of a line.</p>
<p>Geometry and Measurement: Analyzing two- and three-dimensional space and figures by using distance and angle</p> <p>Students use fundamental facts about distance and angles to describe and analyze figures and situations in two- and three-dimensional space and to solve problems, including those with multiple steps. They prove that particular configurations of lines give rise to similar triangles because of the congruent angles created when a transversal cuts parallel lines. Students apply this reasoning about similar triangles to solve a variety of problems, including those that ask them to find heights and distances. They use facts about the angles that are created when a transversal cuts parallel lines to explain why the sum of the measures of the angles in a triangle is 180 degrees, and they apply this fact about triangles to find unknown measures of angles. Students explain why the Pythagorean theorem is valid by using a variety of methods—for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane to measure lengths and analyze polygons and polyhedra.</p>	<p>Data Analysis: Building on their work in previous grades to organize and display data to pose and answer questions, students now see numerical data as an aggregate, which they can often summarize with one or several numbers. In addition to the median, students determine the 25th and 75th percentiles (1st and 3rd quartiles) to obtain information about the spread of data. They may use box-and-whisker plots to convey this information. Students make scatterplots to display bivariate data, and they informally estimate lines of best fit to make and test conjectures.</p>
<p>Data Analysis and Number and Operations and Algebra: Analyzing and summarizing data sets</p> <p>Students use descriptive statistics, including mean, median, and range, to summarize and compare data sets, and they organize and display data to pose and answer questions. They compare the information provided by the mean and the median and investigate the different effects that changes in data values have on these measures of center. They understand that a measure of center alone does not thoroughly describe a data set because very different data sets can share the same measure of center. Students select the mean or the median as the appropriate measure of center for a given purpose.</p>	<p>Number and Operations: Students use exponents and scientific notation to describe very large and very small numbers. They use square roots when they apply the Pythagorean theorem.</p>